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Method for determining antiphase dynamics in a multimode laser

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We measure the cross spectrum of the intensity fluctuations of pairs of modes for a multilongitudinal mode neodymium-doped yttrium aluminum garnet laser operating in the steady state regime. From the data we build up a picture of how the longitudinal mode fluctuations interfere and directly show the antiphase dynamics of the intensity fluctuations.

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It is well known that antiphase dynamics occurs in a range of nonlinearly coupled systems, for example, Josephson junction arrays [1,2], phase oscillators [3], and lasers (e.g., [4] and references contained therein). Antiphase dynamics is where the cooperative behavior of the system results in cancellation of one or more of the collective modes. The behavior of the total system is then somewhat simpler than might be expected.

Here we report on the use of the cross spectrum of the intensity fluctuations of pairs of cavity modes to study the antiphase dynamics of a multimode laser. The cross spectrum is the Fourier transform of the cross correlation function and is a complex valued quantity. Peaks at the collective mode frequencies are seen in the cross spectrum magnitude and the phase at these frequencies is used to deduce whether the two cavity modes contribute to the collective modes in or out of phase.

We use a multimode solid-state neodymium-doped yttrium aluminum garnet (Nd:YAG) laser in which the relaxation oscillations of the cavity modes comprise an oscillator array that is globally coupled by gain sharing. The laser operates in the steady state regime. For N cavity modes there are N collective mode resonances. These occur in a range of frequencies up to the single mode relaxation oscillation frequency [5]. The frequencies of these resonances are typically of the order of tens of kHz, making their direct study relatively simple. For Nd lasers it takes little effort to excite the collective modes. There are several common excitation methods: examples are the use of ambient noise [6], and modulation of the pump, either continuously [7–9] or by step function [10]. In the work considered in this Rapid Communication we rely on ambient noise to excite the collective modes.

A typical method for studying antiphase dynamics is the power spectrum of intensity fluctuations which for an individual cavity mode shows peaks corresponding to the collective modes, many of which are greatly diminished in the power spectrum of the total intensity [6]. This technique gives no phase information; therefore, this method cannot directly verify the phase clustering as predicted for Josephson junctions [2] and other lasers [10]. One approach that has successfully verified phase clustering is the transfer function technique [9] using modulation of the pump. The information contained in the pole-residue representation of a transfer function describes how each of the cavity modes contributes to each collective mode.

Our Nd:YAG laser (Nd$\sim$1% concentration) is end pumped by the combined beams of two 40 mW diode lasers operating at 808 nm; see Fig. 1. The YAG rod is 10 mm long and 3 mm in diameter. It is perpendicularly cut at the front face and Brewster cut at the other to force linear polarization. This ensures that the laser system does not exhibit polarization instability and switching [11]. The front face of the YAG rod forms one end of the cavity and is highly reflective at 1064 nm but partially reflecting (R$\sim$30%) at 808 nm. Optical isolation (measured to be approximately 35 dB) with a polarizing beam splitter and a Fresnel rhomb was therefore used between the YAG rod and the laser diodes to minimize instabilities in the diodes. The other end of the YAG laser cavity is a spherical mirror (R$\sim$98% at 1064 nm) and radius of curvature 2.5 cm. The output coupling mirror was chosen for relatively low frequency relaxation oscillations rather than high output power. The output power is of the order of 1 mW. The optical length of the cavity is approximately 2.5 cm, giving rise to a measured longitudinal mode spacing of 6.15 GHz. The laser operates on the 1064 nm transition with up to six longitudinal modes, depending on the exact cavity length and pump power. Each longitudinal mode has a good TEM_{00} transverse intensity profile. The output coupler was mounted on a piezoelectric transducer to allow fine control of the cavity length, so that we could offset the laser mode spectrum with respect to the gain spectrum.

To measure the cross spectrum, we recorded simultaneously the intensity of two different laser modes using Fabry-Perot filters to separate them out. Optical isolation

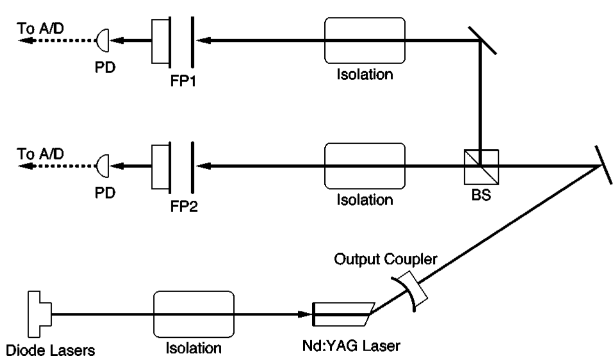

FIG. 1. Schematic of the optical part of the experimental setup. A/D, analog to digital converter; PD, photodiode; FP1, Fabry-Perot 1; FP2, Fabry-Perot 2; BS, 50/50 beam splitter.

*Electronic address: thill@physics.adelaide.edu.au (measured to be approximately 28 dB) with quarter wave plates and polarizing beam splitters was used between the YAG laser and the Fabry-Perot interferometers so that oscillations of collective modes were not driven by feedback. The intensities of the modes transmitted through the Fabry-Perot interferometers were detected with low noise photodiode-amplifiers. The photocurrents were recorded with a 12 bit analog to digital (A/D) converter (Gage Applied Sciences Inc., CompuScope 512) mounted in a PC. Each of the two input channels recorded 2^{19} points at a sample rate of 5 MS/s. The sampling interval was much shorter than the shortest characteristic timescale of the dynamics ($\sim$5 μs) and the number of recorded points gave an adequate amount of data for averaging in the frequency domain. Once the appropriate input range of the A/D was selected, extra gain was applied directly after the photodiode-amplifiers to maximize the signal-to-noise ratio and resolution of the data. The recorded intensity data was transformed into the frequency domain (i.e., power or cross spectrum) using the modified periodigram method [12].

An aluminum spacer was used in the laser cavity. Due to thermal expansion the frequencies of the longitudinal modes and their associated gains could change. One Fabry-Perot (FP1 - TecOptics Ltd., SA-10 with a cavity lifetime of approximately 4.25 ns) was spaced with Invar which has a much lower coefficient of expansion than aluminum. We used it as a reference cavity to which we stabilize one of the laser modes and therefore the laser cavity length. We stabilized by applying a very low level dither to the piezoelectric transducer of FP1 at 3 kHz, which is far enough away in frequency from the collective mode frequencies of the laser not to be a problem. A lock-in amplifier generated an error signal from the intensity of the transmitted laser mode which was fed back to the output coupler. This maintained a sufficiently constant position of the laser cavity modes with respect to the gain spectrum for the time needed to acquire all the data. The other Fabry-Perot (FP2-cavity lifetime of approximately 230 ps) was also dithered and, with a secondary feedback loop, locked to the laser mode it transmitted.

For any pair of cavity modes, the magnitude of the cross spectrum shows peaks at the collective mode frequencies. Zeros (or antiresonances) in the magnitude correspond to phase jumps. Figure 2 shows an example of this for three mode operation. We see three collective mode resonances; i.e., I, II, and III. The labeled feature IV is the second harmonic of the highest frequency collective mode III. The group of features labeled V includes zeros. The phase of the cross spectrum is set so that it is approximately 0° at the frequency of collective mode III. There is a superposed phase shift of approximately 20°/100 kHz due to the amplifier rolloff on the detector used for FP1. The cross spectra for each of the other pairs of modes show peaks at the same frequencies as in Fig. 2; however, the heights of the peaks and their relative phases differ.

At each collective mode frequency the phase has a definite value; that is, it does not jump. We know that the intensity fluctuations of all the laser modes contribute in phase (constructive interference) to the highest frequency collective mode resonance because this is the mode which survives in the total intensity [6,9]. The number of 180° phase jumps between the highest frequency collective mode and any

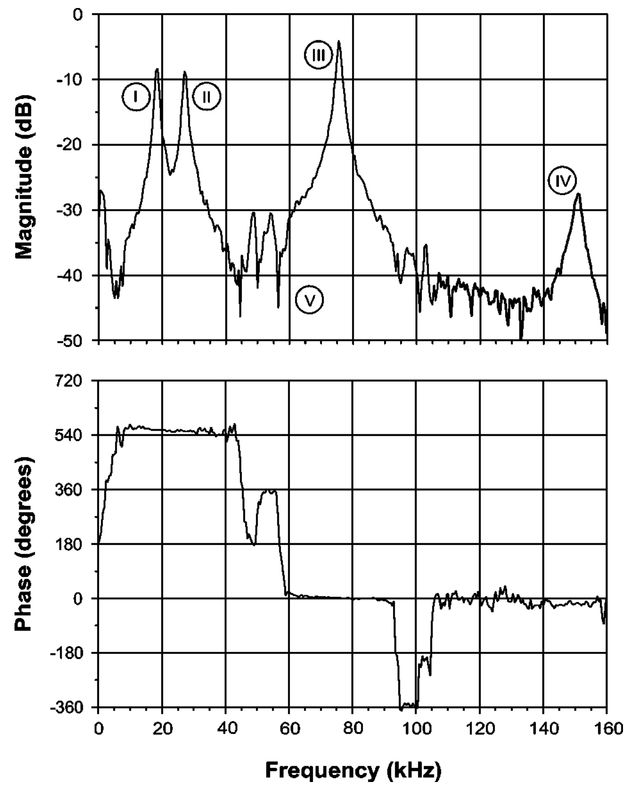

FIG. 2. Cross spectrum of intensity noise fluctuations of cavity modes 1 and 2, for three mode operation. The labeled features in the magnitude are collective modes I, II, and III, at frequencies 18, 26, and 76 kHz, respectively. Feature IV is the second harmonic of mode III, i.e., 152 kHz. The group of features labeled *V* includes zeros associated with phase jumps.

other, tells us whether the fluctuations of each of the two laser modes constructively or destructively interfere at the other collective mode frequencies. That the size of the phase jumps is always a multiple of 180° in this system, implies that we are seeing simple antiphase states and not splay phase states [2]. This is consistent with other experimental data [9] and theoretical work [10,13].

Table I shows the pattern of interferences thus determined for pairs of longitudinal modes. We call this a phase pattern. The collective modes are labeled with Roman numerals in order of increasing frequency, where III is equivalent to the single mode relaxation oscillation frequency [5]. Cavity modes are labeled with Arabic numerals in order of increasing pump threshold. Interference between cavity mode pairs, at a particular collective mode, is labeled ''+'' for in-phase dynamics and ''−'' for antiphase dynamics.

It has been predicted [14] from a nonlinear analysis of the Tang, Statz, and deMars equations (TSdM) [15] that Fabry-Perot lasers may exhibit harmonic and mixing frequency

TABLE I. Phase pattern, expressed in terms of cavity mode pairs, for three mode operation. The columns correspond to the laser cavity mode pairs and the rows to the collective modes.

Mode	1,2	1,3	2,3
I	−	−	+
II	−	+	−
III	+	+	+

TABLE II. Contributions of the cavity modes to the collective modes. The labeling of the modes is the same as in Table I. The column labeled ''Total'' shows the sum of the contributions for the respective rows.

Mode	1	2	3	Total
I	$+0.667$	-0.221	-0.439	$+0.007$
II	$+0.252$	-0.530	$+0.455$	$+0.177$
III	$+0.615$	$+0.643$	$+0.142$	$+1.400$

resonances of the collective modes. For example, in Fig. 2 the labeled feature IV is the second harmonic of collective mode III. However, it is not clear if mixing frequency resonances exist. The two small peaks in the group of features labeled V in Fig. 2 might be such resonances but the signal to noise ratio (at those frequencies) is not good enough to be sure. Also they do not appear consistently in the cross spectra of all possible cavity mode pairs. Another interesting question is whether zeros (of the amplitude), for example, in the group of features labeled V in Fig. 2, are associated with mixing frequencies [16].

To build up a picture of how the cavity modes contribute to the collective modes, we use cavity mode 1, which is the first to reach threshold, as the reference phase. The phases of the contributions of cavity modes 2 and 3 at a given frequency can then be determined from the phase of the cross spectrum between modes 1 and 2, or between 1 and 3 as appropriate. The height of a collective mode peak is the product of the contributions from the two cavity modes concerned. For a given collective mode frequency the heights of that peak in each of the three cross spectra (for three mode operation) can be used to extract the magnitudes of the cavity mode contributions to that collective mode. In obtaining the data the gains of the two detection channels were kept constant between measurements of the time series data for the different pairs of modes, so that this analysis is meaningful. In Table II we show the contributions, with their signs, that we obtained from the set of three cross spectra. The spectrum in Fig. 2 is one of the three. Thus, for collective mode I the fluctuations of cavity modes 2 and 3 are in antiphase with the fluctuation of cavity mode 1. At collective mode II the fluctuations of cavity mode 2 are in antiphase to the fluctuations of cavity modes 1 and 3, while at collective mode III all cavity mode fluctuations are in phase.

The completeness of the destructive interference is another interesting issue. A conjecture has been made [17] that perfect antiphasing will only occur if the modal gains have a symmetric distribution; i.e., the laser gain spectrum is symmetric and the cavity modes are symmetrically disposed about its peak. In a Nd laser the gain spectrum of the 1064 nm line is slightly asymmetric [18]. Furthermore, in our experiment the cavity modes were not symmetrically distributed about the gain peak, so that, according to the conjecture, remnants of the low frequency collective modes will still appear in the power spectrum of the total intensity fluctuations. This imperfect antiphasing was observed experimentally [9] using the transfer function technique, and we also observed this in the experiment reported here. In column 4 of Table II the sums of these contributions are shown. These sums represent the collective mode amplitudes extracted from the cross spectra. For collective mode I this amplitude is quite small, showing that only a vestige of this mode remains. We observed this vestige in the power spectrum of total intensity, which we measured separately.

In this paper we have presented the cross spectrum as a convenient method for detecting what type of antiphasing the dynamics of a nonlinear oscillator array show. If the measured modal fluctuations are sufficiently small, the cross spectral method should give the same information as the transfer function technique [9]. These two techniques would then make it possible to compare the excitation of a nonlinear oscillator array with a single frequency and with noise in the case where nonlinearities are important.

Our method may well be applied to other nonlinear oscillator arrays provided the individual oscillators can be separated. The relaxation oscillation dynamics of a multimode laser is only one manifestation of such an array, albeit a particularly convenient one because the laser cavity modes have different optical frequencies. This makes them separable in the output beam, and thus the cross spectrum at nonoptical frequencies can be measured. Some other optical examples that might be considered are multimode fiber lasers [19], mode-locked lasers, and an array of semiconductor lasers [20]. This technique ought to be applicable in a fairly straightforward manner to the first example. However, the latter two are more complicated, as the cross spectrum at optical frequencies ($\sim 10^{14}$ Hz) will likely be just as important as that near the relaxation oscillation frequency ($10^{5}-10^{10}$ Hz). In the semiconductor laser array, one of the important observables is the ''beam quality,'' which depends on the relative optical phases of the emitters. In this case the individual oscillators in the array might be separated by fiber-coupling of individual emitters in the near-field, or careful imaging of the emitters with a lens to enable separation in the far-field. The cross spectrum might also be a means of probing the dynamics of Josephson junction arrays. Solitons may also be viewed as a nonlinear oscillator array. In this context we note that the cross correlation coefficients between the fluctuation of the spectral components of an intensity squeezed soliton pulse have been measured [21].

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